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TECHNICAL REPORT 231-24

EXPERIMENTS ON TURBULENT WAKES IN A STABLE DENSITY-STRATIFIED ENVIRONMENT

By

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NOTATION

a	Stratification strength, $-\frac{1}{\rho_o}\frac{\partial\rho}{\partial y}$
F, G	Functions
g	Gravitational acceleration
R_i	Richardson number
r_o	Radius of mixed region at $t = 0$ in uniform surroundings
t	Time after generation
t_{col}	Time the maximum thickness is reached
u_o	Velocity of advance of the boundary of a mixed region at $t = 0$ in a uniform medium
v_o	Vertical velocity of advance of the boundary of the mixed region in the vertical direction at $x = 0$ and $t = 0$ in a density-stratified medium
v	Vertical velocity of advance of the boundary of the mixed region in the vertical direction at $x = 0$ in a density-stratified medium
x	Abcissa of boundary of the mixed region at $y = 0$
x_o	Half the horizontal width at $t = 0$
y	Vertical ordinate of the boundary of the mixed region at $x = 0$ in a stratified medium
y_o	Vertical ordinate of the boundary of the mixed region at $x = 0$ and $t = 0$ in a stratified medium
y_{max}	Maximum vertical ordinate of the boundary of the mixed region at $x = 0$ and $t = t_{col}$; half the maximum thickness

y_f	Half the final thickness of mixed region
ϕ	Degree of mixing
ρ	Fluid density
ρ_o	Fluid density at elevation of center of mixed region
$\frac{\partial\rho}{\partial y}$	Density gradient
μ	Dynamic viscosity
D	Diameter of body
V	Velocity of body

ABSTRACT

In a laboratory experiment, turbulent mixed regions were generated in a linearly density-stratified fluid and their behavior was studied. Such regions may occur in nature in the atmosphere and in the ocean. Particularly during their early history, the shape of such regions is influenced by the interacting effects of turbulence and buoyancy, culminating in the occurrence of a maximum thickness and subsequent vertical collapse. A Richardson number (equivalent to the ratio of the characteristic turbulence time and the Vaisala period) was found satisfactorily to correlate the data obtained, together with those previously obtained by other investigators with self-propelled bodies. An estimate is made of the degree of mixing that takes place inside a turbulent mixed region during its growth in stably-stratified surroundings; the effectiveness of this mixing determines the ultimate thickness to which the mixing region collapses.

INTRODUCTION

Stable density stratifications commonly exist in the atmosphere as well as in the ocean, and mixed turbulent regions are created in these media in a number of ways, for example, mixed patches produced by over-turning internal waves or the remnants of chimney trails or thermals in the atmosphere. Also bodies traveling through such media produce a quite different turbulent wake behind them than in a homogeneous medium. The present study is concerned with the time history of a slice of such a wake from its generation onward, which is closely equivalent to studying the change of wake shape with distance behind the body.

In homogeneous surroundings, a turbulent wake grows in size due to turbulent diffusion at the boundary, the rate of spreading being directly proportional to the turbulent intensity. Due to spreading and turbulence decay, the turbulent intensity and consequently the spreading rate continuously decrease with time. In a stratified fluid, turbulent diffusion produces a density distribution inside the turbulent wake different from that of the surrounding fluid. At the same time buoyancy forces interact with the turbulence causing changes in the latter, see Webster (1964), for example. As the turbulent intensity decreases and the stabilizing buoyancy forces increase with time, a condition is reached wherein turbulent diffusion in the vertical direction is completely suppressed. At this instant, the wake or mixed region has reached a maximum vertical thickness. If the fluid inside the region is completely mixed and non-turbulent, all the interior fluid will flow to the level of the center of the wake, and with time the thickness becomes thinner and thinner, as was shown by Wu (1969). With incomplete turbulent mixing, the fluid inside will not have a uniform density and only a partial vertical collapse occurs as was previously observed by Schooley and Stewart (1963) and by Kennedy, Stockhausen and Clark (1966).

Except for the final stage the history of a turbulent wake in a density-stratified medium is shown herein to be governed by the Richardson number, a dimensionless combination of the initial turbulent intensity (actually, the initial spreading velocity of the wake), the density stratification of the medium, gravity, and the initial wake radius. The interesting parameters such as maximum thickness, time necessary to reach this maximum, and final thickness, are all seen to depend upon the value of this Richardson number.

The data for the present study are taken from experiments carried out by van de Watering (1966). His experiments and previous experimental results are reanalyzed herein. The present analysis not only explains, but even enables a prediction of the changes in cross-sectional shape with time, which are experienced by a turbulent mixed region in any linearly density-stratified medium.

INITIAL CONSIDERATIONS

The present study is concerned with the history of the cross-sectional shape of a turbulent wake (having an initial radius r_o and a spreading rate u_o in a homogeneous medium of density ρ_o and viscosity μ), produced by a body which can be characterized by a diameter D and which travels with a velocity V through a density-stratified medium of density ρ_o at the height of the vehicle, of viscosity μ, and density gradient $\partial\rho/\partial y$, while the gravitational acceleration is g. Newton's law for a fluid particle of volume Q and density ρ_o, which is $\Delta\rho$ heavier than its surrounding fluid, states that the gravitational force, acting on this particle, is:

$$\Delta\rho g Q = (\text{mass}) \times (\text{acc.}) = (\rho_o Q) \times (\text{acc.}) \qquad [1]$$

From [1] the acceleration, denoted by g_* , is equal to

$$\text{acc.} = g_* = \frac{\Delta\rho}{\rho_o} g$$

For a linearly density-stratified fluid this acceleration is:

$$\frac{d^2y}{dt^2} = g_* = \frac{- y \frac{\partial\rho}{\partial y} g}{\rho_o} \qquad [2]$$

where y is the vertical distance from the particle to its equilibrium position.

In the present context r_o is chosen as the length scale in [2]:

$$g_* = \frac{-r_o \frac{\partial\rho}{\partial y} g}{\rho_o} = r_o a g$$

From [2] it can be derived that the expression $\sqrt{\frac{-1}{\rho_o} \frac{\partial\rho}{\partial y} g}$ is the frequency at which a displaced particle, if released, will oscillate around its equilibrium position in a linearly stratified fluid. It is known as the Vaisala-Brunt frequency. We will refer to it as $\sqrt{ag}$.

In stratified fluid the following parameters are of interest: the initial and maximum thickness, $2y_o$ and $2y_{max}$, respectively; the initial wake vertical spreading rate, v_o; the time to reach this maximum thickness, t_{col}, and the final wake thickness, $2y_f$. These dependent variables are a function of the previously listed independent variables:

$$(y_o, v_o, y_{max}, t_{col}, y_f) = F\ (r_o, u_o, D, V, \mu, g_*, \rho_o)$$

Using Buckingham's π-theorem one obtains

$$\left(\frac{y_o}{r_o}, \frac{v_o}{u_o}, \frac{y_{max}}{r_o}, \frac{u_o\ t_{col}}{r_o}, \frac{y_f}{r_o}\right) = G\left(\frac{r_o}{D}, \frac{u_o}{V}, \frac{r_o u_o \rho_o}{\mu}, \frac{u_o^{\,2}}{r_o g_*}\right)$$

The ratio's $\frac{r_o}{D}$ and $\frac{u_o}{V}$ may be determined either by direct measurement or by model tests in wind or water tunnels. The determination of these ratio's is beyond the scope of the present paper, and they are assumed to be known in the following. The third independent dimensionless number is the Reynolds number. Since the flow is always turbulent the Reynolds number does not play a role. The five dimensionless dependent variables are thus seen to be solely a function of the fourth independent dimensionless number, generally known as the densimetric Froude number. Because of the proportionality of velocity of advance of the turbulent front, u_o, and the turbulent intensity, $\sqrt{\overline{u'^2}}$, we may also see this number as a Richardson number, defined in this particular application as:

$$R_i = \frac{r_o\sqrt{ag}}{u_o}$$

For the above reasons we may thus expect, that the five dimensionless parameters of interest will vary only if the Richardson number is varied. The Richardson number has been used before to scale flows in which turbulence and gravity forces are interacting (see e.g. Townsend, 1958, or Defant, 1961). It may also be thought of as the ratio of a characteristic turbulence time, r_o/u_o to a characteristic stratification time, $1/\sqrt{ag}$.

Through multiplication of two dimensionless numbers one obtains $t_{col}\sqrt{ag}$, which is to be preferred above $t_{col}u_o/r_o$ to nondimensionalize the time it takes to reach the maximum vertical thickness.

Next let us see how the laboratory experiments prove that indeed the pertinent parameters associated with turbulent wake growth and collapse only depend upon the value of the Richardson number.

EQUIPMENT AND EXPERIMENTAL TECHNIQUE

The spatial behavior of a turbulent wake in a density-stratified fluid is believed closely equivalent to the time history of a thin slice of such a wake, considered as two-dimensional. With this analogy in mind we designed the experimental equipment and technique.

The experiments were carried out in a transparent lucite tank, 121 x 109 x 31.5 cm, in which a linear density stratification was obtained by mixing increasing amounts of sodium chloride in water and introducing these mixtures into the tank in layers through a diffusor placed on the bottom of the tank. The layers were 2.54 cm thick and alternately colored with red dye. In this way 24 layers were brought into the tank which formed initially a stepped density distribution. Overnight a linear density gradient was obtained through molecular diffusion, but the dye remained in discrete layers because of its relatively low diffusivity.

At mid-depth of this linearly density-stratified fluid, a spiral paddle was supported across the tank by a 0.8 cm I.D. brass tube and rotated forward and back by means of a pendulum type of arrangement at the back of the tank. Four rows of holes were drilled along this tube. Just prior to a test, a rod, heavily painted with a blue dye which rapidly dissolves in water, was inserted in the perforated center tube. The blue dye was used to facilitate visualization of the turbulent mixed region. The above-described technique to generate a turbulent mixed region was first used by Wu in 1965. Two spiral paddles were used in these experiments; one was 5.08 cm, the other 10.16 cm in diameter (see Figure 1). With the 5.08 cm paddle, two different pendulums were used so as to vary the turbulent mixing characteristics.

The life of the mixed region was photographed with a 16 mm movie camera. Zero time was taken at the instant the paddle motion stopped. Data were taken from tracings of projected frames of the movie film (see Figure 2).

If desired, more information pertaining to the experimental technique and apparatus can be obtained from the original data report by van de Watering (1966).

RESULTS

With the three combinations of paddles and pendulum, tests in pure water were carried out first. The cross-section of the mixed region during these tests remained more or less circular. From tracings, the averaged (root-mean-square) radius was determined and plotted versus time. The initial velocity of advance of the turbulent front in pure water, u_o, together with the initial averaged radius of the turbulent region r_o, are given in Table 1.

In density-stratified surroundings, the shape of the mixed region changes continuously. Just after generation, the cross-section is already non-circular and the density inside the mixed region is different from its surrounding medium, causing stabilizing gravitational forces to counteract turbulence. Webster (1964) and others clearly show that the vertical scale of turbulence is reduced. With increasing time the cross-section becomes more ellipse-like and eventually reaches a maximum thickness, after which a vertical collapse takes place to a finite thickness, $2y_f$. A series of sample pictures is presented in Figure 2, in which the above-mentioned characteristics are clearly visible. For a typical time history see also Figure 3.

Since one of the primary aims of the present study was to relate turbulent mixing and density gradient with the characteristics of the mixed region, most attention was focused on the change with time of the vertical ordinate of the boundary of the mixed region at the center of the wake, since it is this thickness which is affected most by the interplay of turbulence and gravity. For several runs the horizontal extent of the mixed region was also analyzed in order to obtain information on the rate of collapse. For all tests, the actual measured values for half the maximum thickness (y_{max}), the time it took to reach this maximum (t_{col}) and half the final thickness (y_f) are given in Table 1.

ANALYSIS AND DISCUSSION

In a stably stratified fluid, the vertical ordinate and velocity of the boundary of the mixed region, both taken at the center of the wake and given the symbols y and v respectively, are affected by the interaction of turbulence and density stratification and by the length scale. The experiments indicate that this interaction is already effective during the generation stage. The data show, see Table 1, that at zero time, y_o, and v_o are in almost all tests smaller than their corresponding values in pure water, r_o and u_o. In Figure 4 the dimensionless initial vertical ordinate and velocity are plotted versus the Richardson number. We have included the data points from the two studies with self-propelled bodies, the experiments by Schooley and Stewart (1963) and by Kennedy, Stockhausen and Clark (1966). From the former study, the values for r_o and u_o were obtained from the original movie film of these authors, i.e., from tracings of projected frames of their film of the two runs in pure water. At this point, it may be mentioned that in their analysis, a small error was made in the calculations of the density gradient. The corrected value for $\sqrt{ag}$ is 2.19 sec^{-1} while u_o was found to be 5.87 cm/sec and r_o equal to 0.568 cm. For the experiment carried out by Kennedy, et al, the correct Vaisala-Brunt frequency is: $\sqrt{ag} = 0.25$ sec^{-1}. The values for u_o and r_o were obtained from the curve of the horizontal width of the wake, which is not significantly affected by gravitational effects until collapse starts. The values obtained from these studies are tabulated in Table 1.

The velocity seems much more affected by the interaction than the thickness, see Figure 4. The scatter of the data does not justify a definite relation, but the tendencies are clear. For increasing Richardson numbers, (corresponding to a large stratification), both y_o/r_o and v_o/u_o tend to be reduced, while for small Richardson numbers (turbulence is large compared to stratification), in the range from 0 to 1, there is no significant effect. In the ocean, R_i may vary from about 0.06 to 0.9

and we may thus safely say that in this range the effect of stratification upon initial conditions is negligible for all practical applications.

During the growth stage the buoyancy forces increase while the turbulence decreases in intensity. The increasing dominance of buoyancy culminates at some time after generation, t_{col}, in a maximum wake thickness ($=2y_{max}$). The maximum thickness, y_{max}/r_o, where r_o is the radius of the circular mixed region in uniform surroundings at time zero, is plotted in Figure 5a. In all laboratory experiments on turbulent wake collapse, the maximum thickness can be observed and measured rather accurately. Due to the nature of the variation of thickness with time it is much more difficult accurately to determine the time at which this maximum occurs. Therefore, we may expect more scatter in the data of t_{col} as compared to those of y_{max}. The data are presented in Figure 5b. It is not possible accurately to determine the time at which collapse started from the experimental results as given by Kennedy and his co-workers. Therefore, the possible range of $t_{col} \sqrt{ag}$ is indicated.

During the growth stage, the turbulent mixing inside the wake will produce therein a density gradient less pronounced than that of the surrounding fluid. After the wake grows to its maximum thickness, the fluid particles within seek their equilibrium level under the action of hydrostatic pressures. The mixed region decreases in thickness and spreads rapidly horizontally. This phenomenon is known as wake collapse (Schooley and Stewart, 1963 and Schooley, 1967). If the mixed region is non-turbulent and fully mixed, a long thin wedge will be formed, which becomes thinner with increasing time; such well mixed regions have been observed and studied by Wu (1969). Inside the usual turbulent mixed region in a stratified fluid, turbulent diffusion does not succeed in mixing completely the wake contents, and it is for this reason that there exists an effective density distribution within the wake at the time when the maximum thickness is reached. This density distribution inside the mixed region should again depend only upon the Richardson number.

Wake collapse is driven by the hydrostatic pressure resulting from the differences between the exterior (ambient) and interior density gradients. The interior gradient is continually increased as a result of the vertical shrinking of the wake. The mixed region thus reaches a final thickness, ($2y_f$), corresponding to which these density gradients are of equal value. The measured values of y_f are given in Table 1 and plotted dimensionlessly in Figure 5c.

The trend of the data, shown in Figures 5a, 5b and 5c, is readily explained. A turbulent mixed region in homogeneous fluid ($R_i = 0$) will continue to grow indefinitely in size and y_{max}, t_{col} and y_f are unbounded at this limit. On the other hand the non-turbulent mixed region (Wu, 1969) in density-stratified fluid ($u_o = 0$ and $R_i \rightarrow \infty$) is the other limit for which we know that there is no growth, i.e., $t_{col} = 0$, while y_f approaches zero. In between these limits the data show a gradual increase in the value of the parameters as the Richardson number decreases, i.e. as the wake becomes relatively more turbulent. The opposite case, i.e. decreasing maximum thickness, time to reach maximum thickness, and final thickness, is observed when the density gradient or R_1 is increased. This interpretation also explains why it was observed that with the larger paddle (larger R_1) the turbulent mixed region did not seem to have much strength i.e. the maximum thickness was small compared with the initial diameter in a homogeneous medium.

In the past, the time t_{col} has generally been considered as solely dependent upon the density gradient (see for instance Schooley, 1967 and 1968), but the present analysis shows that it is the Richardson number that actually governs the value of $t_{col}\sqrt{ag}$.

Let us suppose as an approximation that at $t = t_{col}$ a linear density gradient $(\partial\rho/\partial y)_{internal}$ exists inside the mixed region, see Figure 6. The difference in density of the fluid at the top of the wake, (1) in Figure 6, compared to fluid in the center (2), is:

$$\rho_1 - \rho_o = y_{max} \cdot (\partial\rho/\partial y)_{internal}$$

If we assume that no further mixing takes place after t_{col}, then finally:

$$y_{max} (\partial\rho/\partial y)_{internal} = y_f (\partial\rho/\partial y)_{external}$$

So that,

$$\frac{y_f}{y_{max}} = \frac{\left(\frac{\partial \rho}{\partial y}\right)_{\text{internal at } t_{col}}}{\left(\frac{\partial \rho}{\partial y}\right)_{\text{external}}} \qquad [3]$$

The right hand side of this expression is a measure for the degree of mixing, Φ, which takes place during growth and which we define as:

$$\Phi = \left[1 - \frac{\left(\frac{\partial \rho}{\partial y}\right)_{\text{internal at } t_{col}}}{\left(\frac{\partial \rho}{\partial y}\right)_{\text{external}}}\right] \qquad [4]$$

expressed in percent. Combining [3] and [4], the degree of mixing is:

$$\Phi = \left[1 - \frac{y_f}{y_{max}}\right] \times 100 \qquad [5]$$

In Figure 7 the values of y_f/y_{max} and Φ are plotted versus the Richardson number. Increasing the density gradient (i.e., increasing the value of the Richardson number) will result in less mixing and from Figure 7 it is seen that indeed the degree of mixing decreases with increasing Richardson number. Mixing is generally poor: the degree of mixing, Φ, is about 40 percent at the smallest Richardson numbers and decreases if R_i increases.

As far as horizontal wake spreading is concerned, it should be realized first that a turbulent wake expands naturally unless opposed, as by hydrostatic pressures. While stratification acts against expansion in the vertical direction, it actually enhances the horizontal expansion. Initially the horizontal expansion is dominated by turbulence and the effect of stratification is small. Consequently the horizontal expansion until collapse sets in, will be close to the pure water case. However, once, the maximum vertical thickness is reached, the wake collapses vertically and the fluid spreads out horizontally under the action of hydrostatic pressures. We may thus not be surprised to observe a sudden surge in horizontal expansion some short time after collapse sets in. At this time gravitational forces start to play a completely dominant role. For several runs the horizontal spreading is plotted dimensionlessly in Figure 8. A distinct discontinuity in the x-t curves is noticeable and it occurs in each case a short time after the maximum thickness is reached.

In experiments with a non-turbulent mixed region, the area of mixed fluid remains constant throughout a test. In a turbulent mixed region, an increase in cross-sectional area with time due to turbulent entrainment is observed. In stratified fluid, we observed turbulent entrainment even after the maximum thickness was reached. A continuous increase in area was measured, although the rate of area increase decreased gradually with time.

The vertical collapse of the mixed region generated internal waves in the surrounding fluid due to displacement of mixed fluid by the ambient medium. In a turbulent mixed region, even after the onset of collapse, there is still turbulence, albeit that the motion is dominated by gravitational effects. This turbulence, which tends to expand the mixed region, tends to soften wake collapse. The net result is to increase the characteristic time of wake collapse over that observed in the case of the collapse of a non-turbulent mixed region. This time is further increased because of the density gradient existing within the wake. Furthermore, in the turbulent case the mixed region typically collapses to a final thickness which is a large fraction of the maximum thickness. This is shown in Figure 7. The punch action of the impulsive collapse is therefore not only slower in time, but also of a smaller amplitude. Collapsing turbulent mixed regions are consequently less effective generators of internal waves than non-turbulent completely mixed regions. All of these effects were observed in the present experiments.

Two other interesting features were also observed which seem to be characteristic for a collapsing turbulent mixed region. After reaching its maximum thickness, mixed fluid spreads out horizontally and the upper and lower wake boundaries become more or less flat; this was particularly clear near and after the time when the final thickness was reached. Simultaneously, "finger-like" puffs of mixed fluid began protruding horizontally away from the bulk of the wake. Presumably these phenomena are due to the action in the large turbulent eddies, which are suppressed and flattened by the stabilizing buoyancy forces.

CONCLUSIONS

A turbulent mixed region in a density-stratified fluid undergoes changes as a result of the interplay between turbulence and stabilizing buoyancy forces. Some time after generation the mixed region reaches a maximum vertical thickness, after which vertical collapse and horizontal spreading take place. Due to incomplete mixing, the density of the fluid inside the region is not uniform, and, consequently, the mixed fluid reaches an equilibrium shape with a finite vertical thickness. Based on the present experimental results, obtained from tests using paddle mixers as generators of turbulent mixed regions, and on other available data from previous studies by other investigators, the maximum thickness, the time it takes to reach this maximum, and the final thickness, are shown to depend upon the ratio of the characteristic turbulence time to the Vaisala-Brunt period.

This ratio is in the form of a Richardson number. The present correlation provides an explanation for the higher values of the maximum thickness and the time to reach this maximum, as obtained by other investigators with self-propelled bodies under conditions of relatively small Richardson numbers.

A rough picture of turbulent wake collapse is presented with which, together with a knowledge of the ratio of final to maximum thickness (which can be estimated from the present correlation), the degree of mixing inside the wake can be estimated. Mixing is generally poor, in the order of 10 to 40 percent (in a completely mixed fluid the mixing is 100 percent).

The collapse of a turbulent mixed region in a density-stratified medium generates less pronounced internal waves in comparison with a non-turbulent, completely mixed region.

The present experiments revealed the formation of a flat top and bottom of the mixed region and of "finger-like" puffs of mixed fluid spreading horizontally outward.

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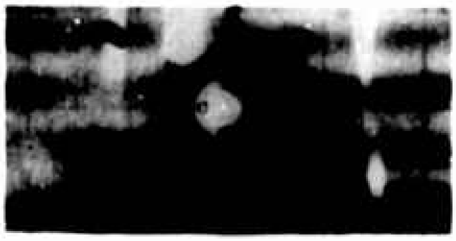
TABLE 1

Investigators	r_o, y_o cm	u_o, v_o cm/sec	$\sqrt{ag}$ sec^{-1}	$\frac{r_o\sqrt{ag}}{u_o}$	t_{col} sec	$t_{col}\sqrt{ag}$	y_{max} cm	y_{max}/r_o	y_{final} cm	$\frac{y_{final}}{r_o}$
van de Watering 1966	4.83	0.890	0.000	0.00	∞		∞	∞	∞	∞
	4.64	0.890	0.254	1.34	5.25	1.33	7.26	1.51	5.36	1.110
	3.96	0.763	0.296	1.56	5.00	1.48	6.03	1.25	5.03	1.041
	4.35	0.813	0.296	1.56	4.10	1.21	6.33	1.31	5.61	1.162
	3.81	0.701	0.415	2.19	3.25	1.35	5.36	1.11	4.34	0.898
	3.91	0.767	0.415	2.19	3.25	1.35	5.74	1.19	4.35	0.900
	4.08	0.620	0.585	3.09	2.40	1.40	5.10	1.06	--	--
	4.07	0.432	0.820	4.33	1.50	1.23	4.59	0.95	3.74	0.774
	4.14	0.610	0.810	4.27	1.30	1.05	4.83	1.00	4.02	0.832
	4.50	1.170	0.000	0.00	∞		∞	∞	∞	∞
	4.06	0.915	0.408	1.56	2.75	1.12	5.56	1.24	4.22	0.937
	3.73	0.889	0.408	1.57	2.75	1.12	5.62	1.25	3.99	0.886
	4.14	0.871	0.593	2.28	1.50	0.89	5.38	1.20	4.50	1.000
	4.22	0.686	0.833	3.21	1.50	1.25	5.08	1.13	4.22	0.937
	7.81	1.740	0.000	0.00	∞		∞	∞	∞	∞
	8.18	0.661	0.420	2.06	2.52	1.05	9.35	1.20	8.00	1.025
	7.11	0.648	0.418	1.89	1.75	0.73	8.15	1.04	6.88	0.881
	7.52	0.787	0.814	3.69	1.30	1.07	8.18	1.04	--	--
	6.30	1.068	1.165	5.21	1.10	1.28	6.91	0.88	6.00	0.769
	6.71	0.839	1.181	5.29	1.00	1.18	7.42	0.95	6.84	0.876
Kennedy, Clark and Stockhausen 1966	7.78	4.14	0.00	0.00	∞		∞	∞	∞	∞
	7.78	4.14	0.25	0.47	6.5-13	1.63-3.25	17.17	2.12	11.88	1.516
Schooley and Stewart 1963	0.568	5.87	0.00	0.000	∞		∞	∞	∞	∞
	0.568	5.87	2.19	0.212	0.97	2.12	2.375	4.18	1.5	2.64

FIGURE 1 - SPIRAL PADDLES; 10.16 cm AND 5.08 cm IN DIAMETER



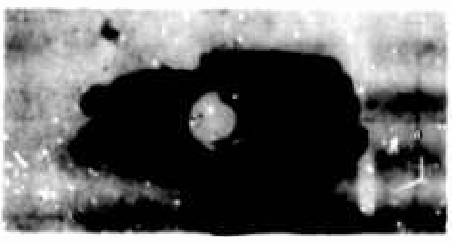
$t\sqrt{ag} = 0$

(JUST AFTER GENERATION)

$t\sqrt{ag} = 1.33$

(MAXIMUM THICKNESS)

$\sqrt{ag} = 0.254\ \mathrm{sec}^{-1}$



$t\sqrt{ag} = 4.10$

$t\sqrt{ag} = 7.00$

FIGURE 2 - SAMPLE PICTURES OF TURBULENT MIXED REGION IN STRATIFIED FLUID

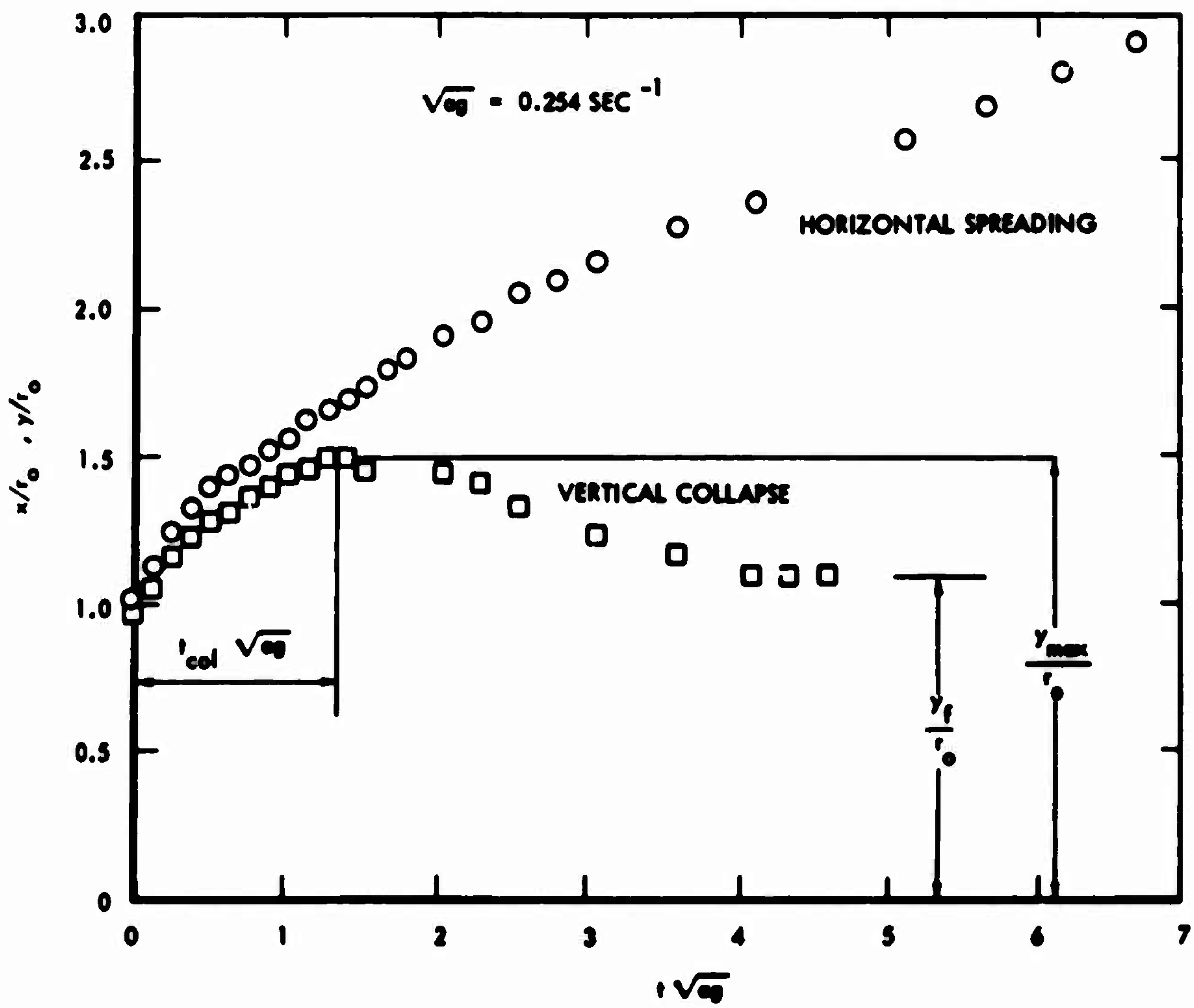


FIGURE 3 - TYPICAL HISTORY OF A TURBULENT MIXED REGION IN A DENSITY - STRATIFIED MEDIUM.

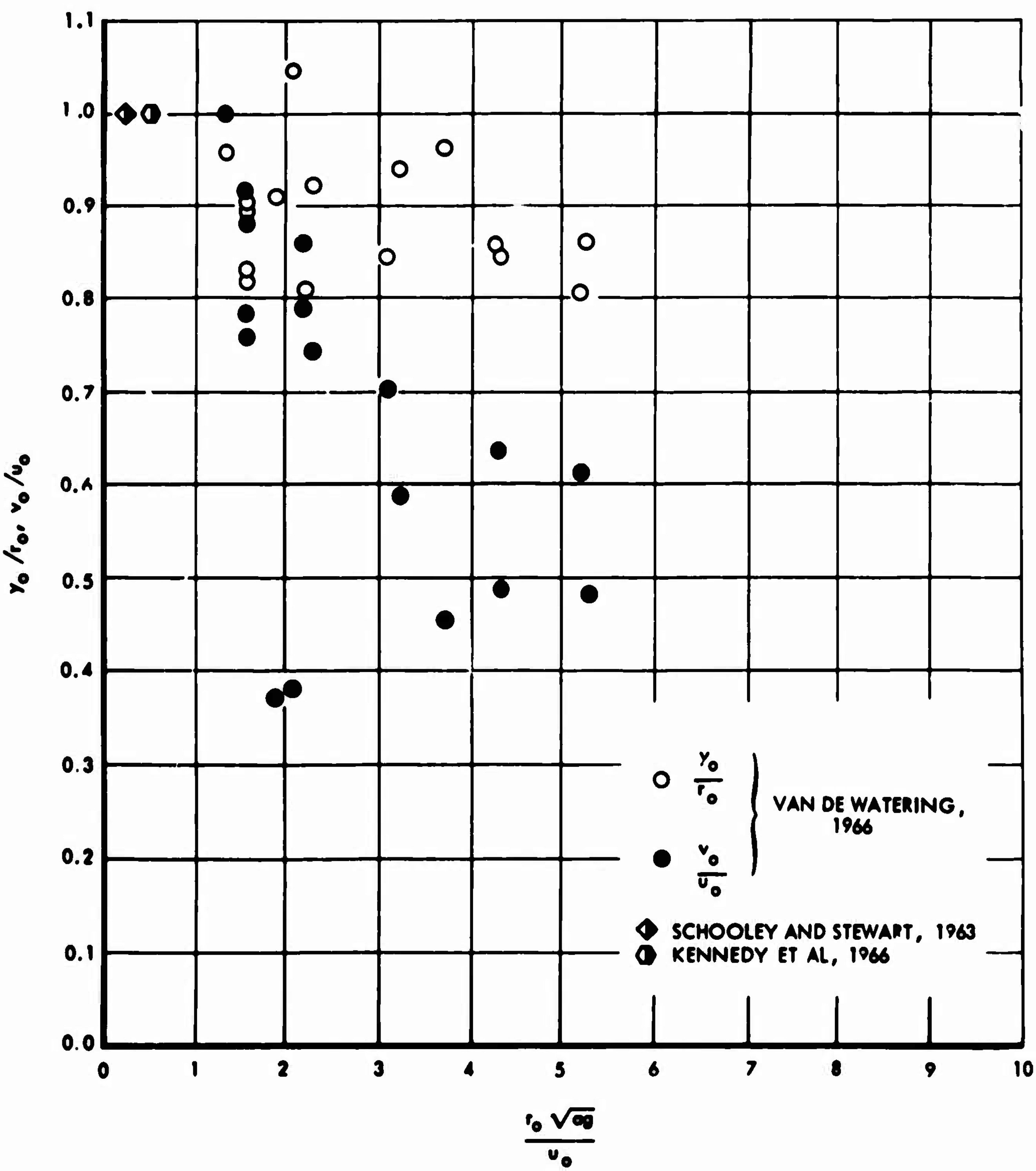


FIGURE 4 - DEPENDENCE OF INITIAL CONDITIONS UPON RICHARDSON NUMBER.

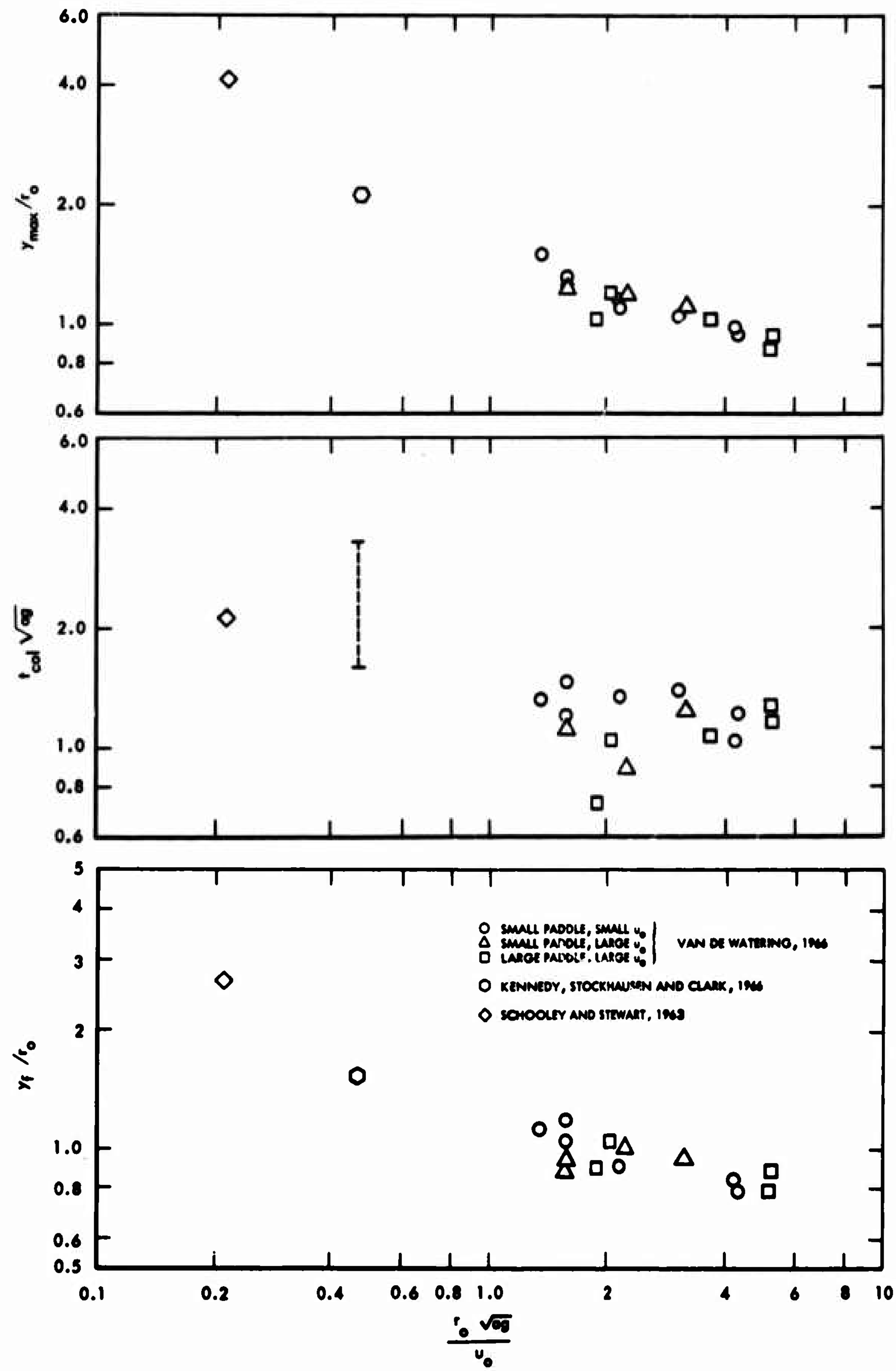


FIGURE 5 - CORRELATION OF MAXIMUM THICKNESS, THE TIME MAXIMUM THICKNESS IS REACHED AND THE FINAL THICKNESS WITH THE RICHARDSON NUMBER.

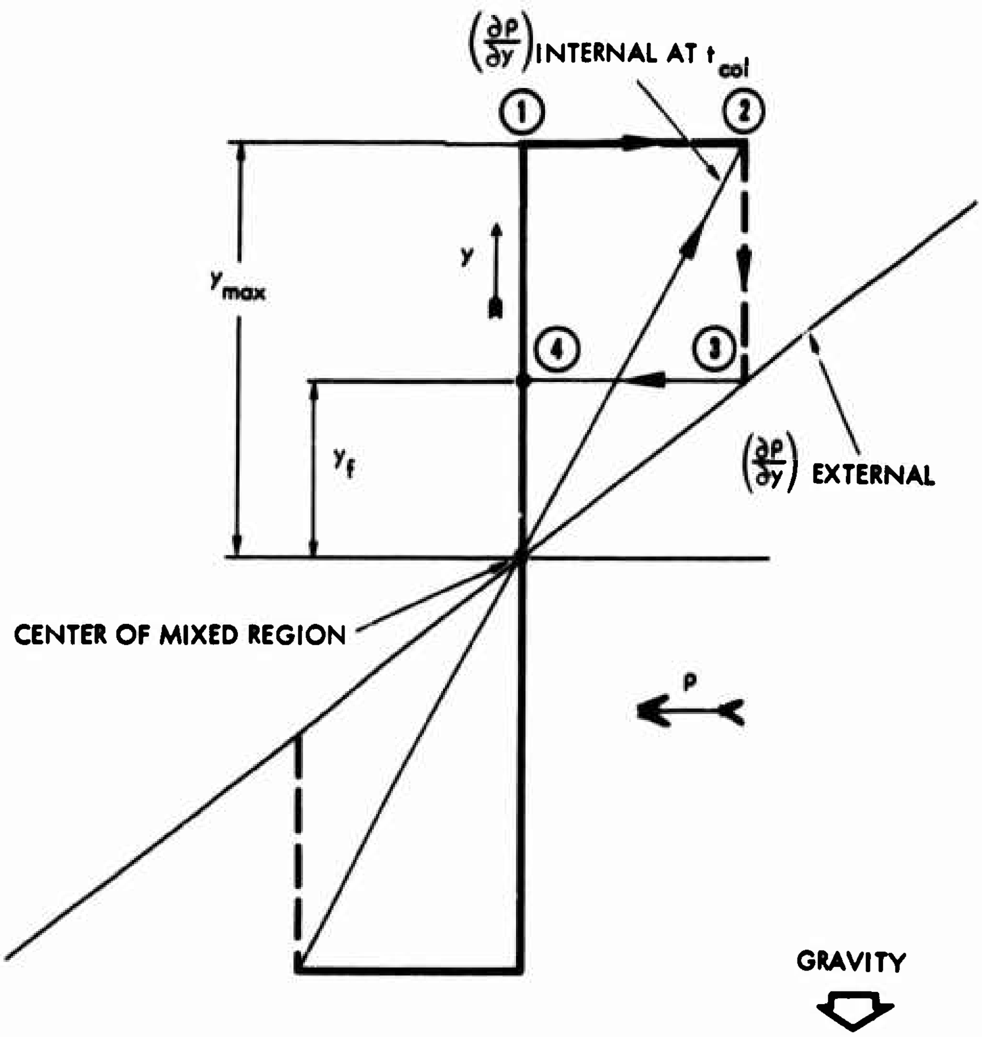


FIGURE 6 - GRAPHICAL DETERMINATION OF FINAL THICKNESS

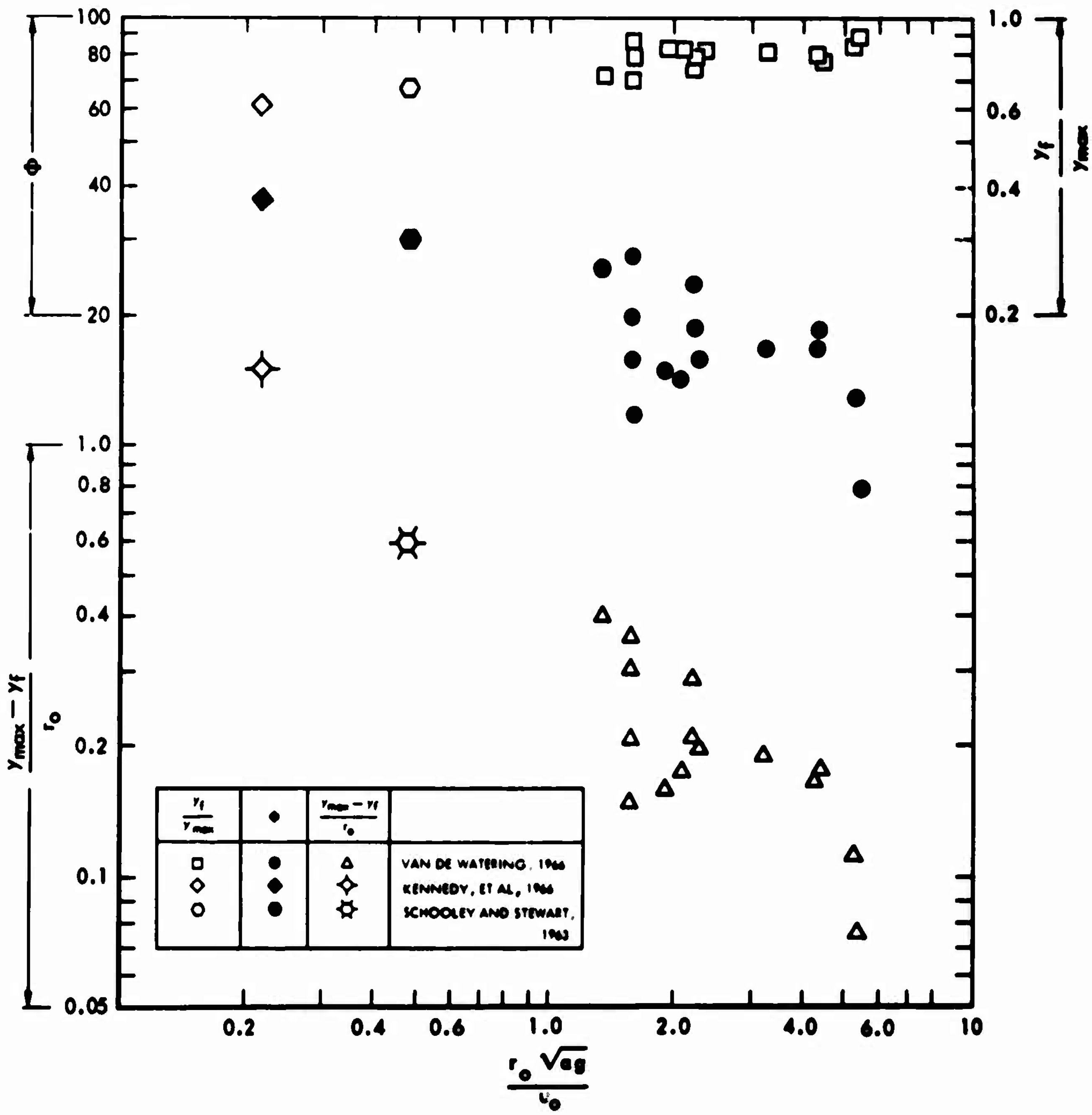


FIGURE 7 - CORRELATION OF MAXIMUM THICKNESS, FINAL THICKNESS AND DEGREE OF MIXING WITH THE RICHARDSON NUMBER.

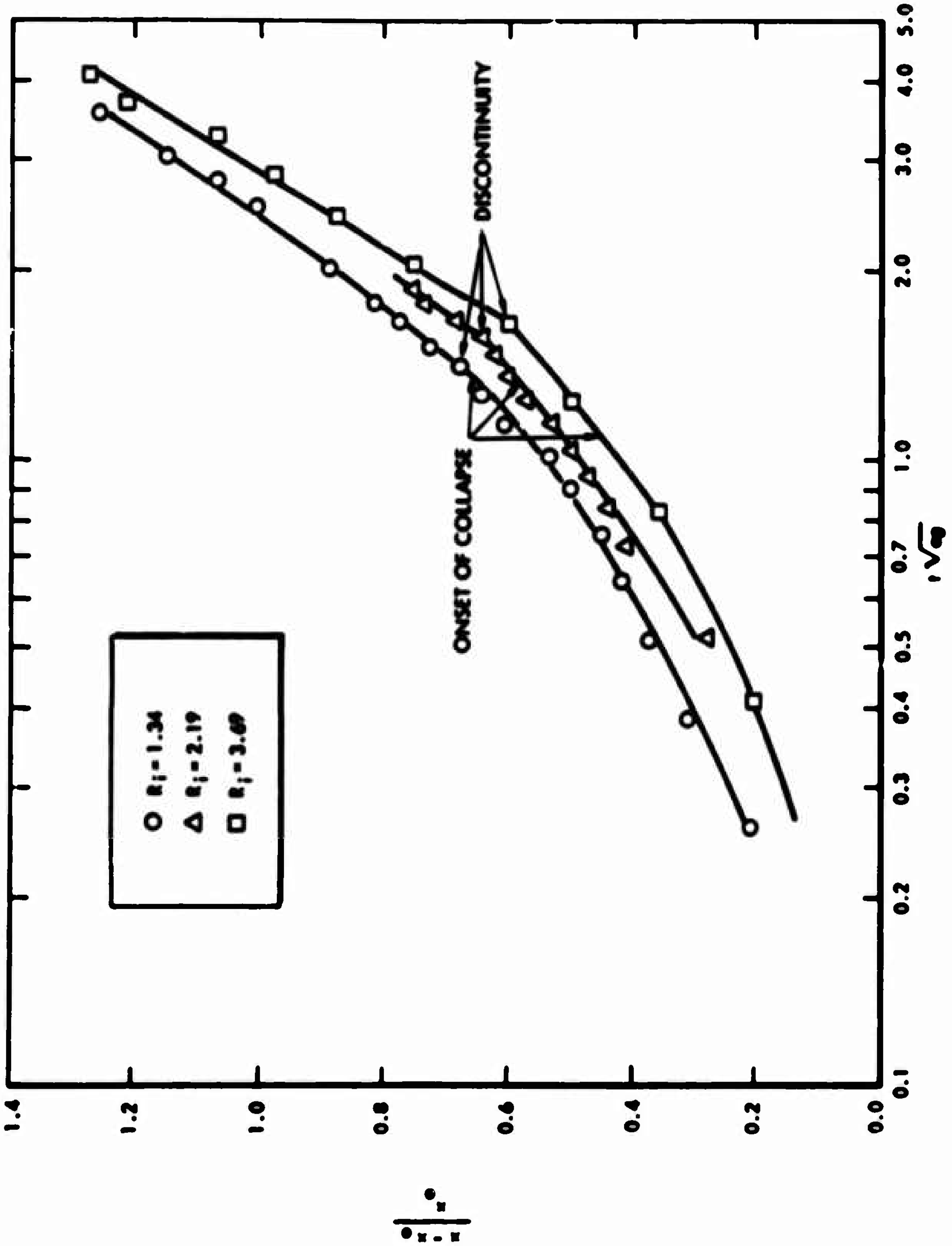


FIGURE 8 - HORIZONTAL SPREADING BEFORE AND AFTER ONSET OF COLLAPSE

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13. ABSTRACT

In a laboratory experiment, turbulent mixed regions were generated in a linearly density-stratified fluid and their behavior was studied. Such regions may occur in nature in the atmosphere and in the ocean. Particularly during their early history, the shape of such regions is influenced by the interacting effects of turbulence and buoyancy, culminating in the occurrence of a maximum thickness and subsequent vertical collapse. A Richardson number (equivalent to the ratio of the characteristic turbulence time and the Vaisala period) was found satisfactorily to correlate the data obtained, together with those previously obtained by other investigators with self-propelled bodies. An estimate is made of the degree of mixing that takes place inside a turbulent mixed region during its growth in stably-stratified surroundings; the effectiveness of this mixing determines the ultimate thickness to which the mixing region collapses.

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14. KEY WORDS	LINK A ROLE	LINK A WT	LINK B ROLE	LINK B WT	LINK C ROLE	LINK C WT
Turbulent Wake in Stratified Fluid						
Wake Growth						
Wake Collapse						
Turbulence Suppression						
Stratified Fluid						
Continuous Density Gradient						
Turbulence						
Richardson Number						
Turbulent Mixing						
Internal Waves						

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